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4 Sets

4.1 Sets

Definition 4.1. A *set* is an unordered collection of things without repetitions. The things in set S are called the *members* of S.

Definition 4.2. A *set enumeration* is one way to describe a set, by writing the members of the set in braces, separated by commas. For example, {2, 5, 9} is a set of three integers.

4.1.1 Finite and infinite sets

It is possible to list the members of a *finite* set. But some sets, such as the set of all positive integers, have infinitely many members. Here are a few common infinite sets.

$\mathcal{N}$	$\{0, 1, 2, 3, \ldots\}$
$\mathcal{Z}$	$\{\ldots, -2, -1, 0, 1, 2, \ldots\}$
$\mathcal{R}$	the set of all real numbers

4.1.2 Set comprehensions

A *set comprehension* is a way to describe the set of all values that have a certain property. Notation

$$\{x \mid p(x)\}$$

stands for the set of all values x such that $p(x)$ is true and notation

$$\{f(x) \mid p(x)\}$$

stands for the set of all values $f(x)$ such that $p(x)$ is true. Notation

$$\{x \in S \mid p(x)\}$$

is shorthand for $\{x \mid x \in S \wedge p(x)\}$ Here are some examples.

Set	Description
$\{x \mid x \in \mathcal{R} \ \wedge \ x^2 - 2x + 1 = 0\}$	$\{-1,\ 1\}$
$\{x \in \mathcal{R} \mid x^2 - 2x + 1 = 0\}$	$\{-1,\ 1\}$
$\{x \mid x$ is an even positive integer$\}$	$\{2, 4, 6, \ldots\}$
$\{x^2 \mid x$ is an even positive integer$\}$	$\{4, 16, 36, \ldots\}$

4.1.3 Set notation and operations

Table 4-1 defines notation for sets.

4.1.4 Identities for sets

Table 4-2 list some identities are easy to establish.

4.1.5 Sets of sets

The members of sets can be sets. For example, if $S = \{\{1, 2, 3\}, \{2, 4, 6\}\}$ then $|S| = 2$, since S has exactly two members, $\{1,\ 2,\ 3\}$ and $\{2,\ 4,\ 6\}$.

Do not confuse $\in$ with $\subseteq$. If $S = \{\{1, 2, 3\}, \{2, 4, 6\}\}$ then

$\{1, 2, 3\} \in S$

$\{1, 2, 3\} \not\subseteq S$

$3 \notin S$

Notice that $\{\} \neq \{\{\}\}$. $|\{\}| = 0$ but $|\{\{\}\}| = 1$ since $\{\{\}\}$ has one member, the empty set.

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Table 4-1	
Notation	**Meaning**
$\|S\|$	$\|S\|$ is the *cardinality* (size) of S, when S is a finite set.
$\{\}$	$\{\}$ is the empty set, which has no members
$x \in S$	$x \in S$ is true if x is a member of set S. For example, $2 \in \{1, 2, 3, 4\}$
$x \notin S$	$x \notin S$ is equivalent to $\neg(x \in S)$
$S \cup T$	$S \cup T = \{x \mid x \in S \vee x \in T\}$. For example, $\{2, 5, 6\} \cup \{2, 3, 7\} = \{2, 3, 5, 6, 7\}$. This is called the *union* of sets S and T.
$S \cap T$	$S \cap T = \{x \mid x \in S \wedge x \in T\}$. For example, $\{2, 5, 6\} \cup \{2, 3, 7\} = \{2\}$. This is called the *intersection* of sets S and T.
$S - T$	$S - T = \{x \mid x \in S \wedge x \notin T\}$. For example, $\{2, 5, 6\} - \{2, 3, 7\} = \{5, 6\}$. This is called the *difference* of sets S and T.
$\overline{S}$	$\overline{S} = U - S$, where U is the domain of discourse. This is called the *complement* of S.
$S \times T$	$S \times T = \{(x, y) \mid x \in S \wedge y \in T\}$. For example, $\{2, 3\} \times \{5, 6\} = \{(2,5), (2,6), (3,5), (3,6)\}$. This is called the *cartesian product* of S and T.
$S \subseteq T$	$S \subseteq T$ is true if $\forall x (x \in S \rightarrow x \in T)$. For example, $\{2, 4, 6\} \subseteq \{1, 2, 3, 4, 5, 6\}$. Notice that $\{2, 4, 6\} \subseteq \{2, 4, 6\}$. $S \subseteq T$ is read "S is a subset of T."
$S = T$	Sets S and T are equal if $S \subseteq T$ and $T \subseteq S$. That is, S and T have exactly the same members.

Table 4-2
Some Set Identities
$A \cup \{\} = A$
$A \cap \{\} = \{\}$
$\overline{\overline{A}} = A$
$A \cup B = B \cup A$
$A \cap B = B \cap A$
$A \cup (B \cup C) = (A \cup B) \cup C$
$A \cap (B \cap C) = (A \cap B) \cap C$
$\overline{A \cup B} = \overline{A} \cap \overline{B}$
$\overline{A \cap B} = \overline{A} \cup \overline{B}$
$A - B = A \cap \overline{B}$.
$A \cup (A \cap B) = A$
$A \cap (A \cup B) = A$