Computer Science 2400
Fall 2021
Practice Quiz 1b
First-Order Logic

Circle the letter of the best answer for each multiple-choice question.

1. Write a clearly legible **T** to the left of each of the following that is true and a clearly legible **F** to the left of each that is false. The domain of discourse is the set of all integers.

   (a) $\exists x(x + x = 1)$

   (b) $\exists x(x + 2 = 1)$

   (c) $\forall x(x^2 - x \neq 1)$

   (d) $\forall x(x^2 - x \neq 0)$

   (e) $\forall x(x^2 > 0)$

   (f) $\exists x(x^2 > 0)$

   (g) $\forall x \forall y(xy = 1)$

   (h) $\exists x \exists y(xy = 1)$

   (i) $\forall x \exists y(xy = 1)$

   (j) $\exists x \forall y(xy = 1)$

   (k) $\exists x \exists y((x + y = x \wedge y \neq 0)$

   (l) $\forall x \forall y((x + y = x \wedge y \neq 0)$

   (m) $\exists x \forall y(xy = y)$

2. Which of the following is equivalent to $\neg \exists x(P(x) \vee Q(x))$

   (a) $\forall x(\neg P(x) \wedge \neg Q(x))$

   (b) $\forall x(\neg P(x) \vee \neg Q(x))$

   (c) $\exists x(\neg P(x) \wedge \neg Q(x))$

   (d) $\exists x(\neg P(x) \vee \neg Q(x))$

3. Suppose

   $P(x)$ means "x showed up with a pencil,"
   $C(x)$ means "x showed up with a calculator."

   Express each of the following in first-order logic using predicates $P(x)$ and $C(x)$.

   (a) At least one of the students showed up with a pencil.

   (b) Every student showed up with a pencil or a calculator (or both).

   (c) Every student who showed up with a calculator also had a pencil.

   (d) There is a student who showed up with both a pencil and a calculator.

   (e) At least two different students showed up with a pencil.

   (f) Exactly one student showed up with a pencil.