CSCI 2405

Answers to Exam 4

Note:

Please do not write an to mean an. You can write a\_n for an, and a\_(n-1) for a­n-1 if you do not want to use subscripts.

If you do not want to use superscripts, use x^n. But be careful about precedence! The ^ operator has high precedence.

 -1^n is not the same as (-1)^n. -1^2 = -1. (-1)^2 = 1.

 x^n-1 is not the same as x^(n-1).

1. A ternary string of length n with 3 consecutive 0s can have the following forms.

 000… (where … is any ternary string of length n-3)

 1… (where … is any ternary string of length n-1 that contains 3 consecutive 0s)

 2… same

 01… (where … is any ternary string of length n-3 that contains 3 consecutive 0s)

 02… same

 001… (where … is any ternary string of length n-3 that contains 3 consecutive 0s)

 002… same

 Notice that 3 initial values are required. A string of length <3 cannot contain 3 consecutive 0s.

 **a0 = 0**

 **a1 = 0**

 **a2 = 0**

 **an = 2an-1 + 2an-2 + 2an-3 + 3n-3 (n ≥ 3)**

 **Notice:**

 **a3 = 1**

 **a4 = 5**

 **a5 = 21**

2. For n > 2, you have two options:

 Start with a dot. Then you have n-1 milliseconds left.

 Start with a dash. Then you have n-3 milliseconds left.

 In 0 milliseconds, you cannot send any messages.

 In 1 millisecond, you can only send one message, a dot

 In 2 milliseconds, you can only send one message, dot dot.

 **a0 = 0**

 **a1 = 1**

 **a2 = 1**

 **a­n = an-1 + an-3  (n > 2)**

3. degree = **2**

4. **an = c4n** where c is a constant. Many students left out c.

5. a) **r2 – 2r – 3 = 0** Parts (a) and (b) were graded together, worth 5 points.

 b) **r = 3 and -1**

 c**) an = c3n + d(-1)n** Do not try to solve for c and d. If you do, you are not

 writing a general solution. It does not matter what you

 call the arbitrary constants c and d.

 d) a0 = 2 = c30 + d(-1)0

 2 = c + d

 a1= 12 = c31 + d(-1)1

 12 = 3c – d

 Adding those two equations gives

 14 = 4c

 c = 7/2

 d = 2 – c = 4/2 – 7/2 = -3/2

 Solution: **an = (7/2)3n – (3/2)(-1)n**

6. a) **r2 – 6r + 9 = 0** Parts (a) and (b) were graded together, worth 5 points.

 b) **r = 3 (with multiplicity 2)**

 c) **an = c3n + dn3n** Do not try to solve for c and d. If you do, you are not

 writing a general solution.

7. an = 2an-1 + 3a­n-2 + 2n

 The associated homogeneous recurrence is

 an = 2an-1 + 3an-2.

 solution from problem 5: an = c3n – d(-1)n

 Since 2 is not a solution of the characteristic equation, the general solution the to

 the nonhomogeneous recurrence is

 **an = c3n + d(-1)n + e2n**

8. an = 2an-1 + 3a­n-2 + 3n

 The associated homogeneous recurrence is

 an = 2an-1 + 3an-2.

 solution from problem 5: an = c3n – d(-1)n

 Since 3 is a solution of the characteristic equation, the general solution is

 **an = c3n + d(-1)n + en3n**

9. an = 2an-1 + 3a­n-2 + n22n

 The associated homogeneous recurrence is

 an = 2an-1 + 3an-2.

 solution from problem 5: an = c3n – d(-1)n

 Since 2 is not a solution of the characteristic equation, the general solution is

 **an = c3n + d(-1)n + en22n + fn2n + g2n**

You can also write it in the form

 **an = c3n + d(-1)n + (en2 + fn + g)2n**

10. a) a = 4, b = 2, d = 1, bd = 2, a > bd, solution **T(n) = ϴ(n2)**

 b) a = 8, b = 2, d = 3, bd = 8, a = bd, solution **T(n) = ϴ(n3log(n))**