2. (a) an = 0 for n < 3

For n > 2, a bit string of length n containing 3 consecutive 0s either

1. starts with 1, which is followed by a string of length n-1 with 3 consecutive 0s,

2. starts with 01, which is followed by a string of length n-2 with 3 consecutive 0s,

3. starts with 001, which is followed by a string of length n-3 with 3 consecutive 0s.

4. starts with 000, which is followed by any string of length n-3 bits,

an = an-1 + an-2 + an-3 +2n-3

(b) a1 = 0

a2 = 0

a3 = 1

a4 = a3 + a2 + a1 + 21

= 1 + 2 = 3

a5 = a4 + a3 + a2 + 22

= 3 + 1 + 4 = 8

a6 = a5 + a4 + a3 + 23

= 8 + 3 + 1 + 8 = 20

a7 = a6 + a5 + a4 + 24

= 20 + 8 + 3 + 16 = 47

3 (a) a1 = 1

a2 = 2

a3 = 4

an = an-1 + an-2 + an-3

(b) a4 = a3 + a2 + a1

= 4 + 2 + 1 = 7

a5 = a4 + a3 + a2

= 7 + 4 + 2 = 13

a6 = a5 + a4 + a3

= 13 + 7 + 4 = 24

a7 = a6 + a5 + a4

= 24 + 13 + 7 = 43

a8 = a7 + a6 + a5

= 43 + 24 + 13

= 81

4 (a) a1 = 0

a2 = 1

A ternary string of length n > 1 either

1. starts with 1 or 2, which is followed by a string of length n-1 with two consecutive 0s

2. starts with 01 or 02, which is followed by a string of length n-2 with two consecutive

0s

3. starts with 00, which is followed by a ternary string of length n-2.

an­ = 2an-1 + 2an-2 + 3n-2

(b) a3 = 2 + 0 + 31 = 5

a4 = 10 + 2 + 32 = 21

a5 = 42 + 10 + 33 = 79

a6 = 158 + 42 + 34 = 281

5. (a) an is only defined for n divisible by 5.

a0 = 1

a5 = 1

an = an-5 + an-10

(b) a10 = 1 + 1 = 2

a15­ = 2 + 1 = 3

a20 = 3 + 2 = 5

a25 = 5 + 3 + 8

a30 = 8 + 5 = 13

a35 = 13 + 8 = 21

a40 = 21 + 13 = 34

a­45 = 34 + 21 = 55

6. Define a\_n = the number of bit strings of length n with an even number 0s

b\_n = the number of bit strings of length n with an odd number of 0s

A bit string of length n containing an even number of 0s either

1. begins with 1, which is followed by a bit string of length n-1 with an even

number of 0s, or

2. begins with 0, which is followed by a bit string of length n-1 with an odd

number of 0s, or

a0 = 1

an = an-1 + bn-1

A similar rule works for defining bn

b0 = 0

b­n = b­n-1 + an-1