**Homework Assignment 4**
**CSCI 2405**
**Spring 2020**
**Section 001**

**Due:** 4/10

1. An office building has 25 floors and 39 offices per floor. How many offices does the building have?

2. How many different 3-letter initials can a person have? This problem and all subsequent ones that refer to letters assume that the alphabet has 26 letters.

3. How many different 3-letter initials are there if none of the letters can be repeated?

4. How many bit strings are there of length 8?

5. How many bit-strings are there that have length $n$ and both start and end with 1?

6. How many strings of four letters contain letter $x$?

7. How many positive integers between 5 and 31 (including 5 and 31) have the following properties? List what the integers are in each case.

   (a) Divisible by 3

   (b) Divisible by 4

   (c) Divisible by 3 and 4

8. How many integers between 1000 and 9999 (including 1000 and 9999)

   (a) are even?

   (b) are divisible by 9?

(c) are not divisible by 3?

(d) are divisible by 3 or 5?

(e) have all different digits when written in base 10?

9. How many strings of 4 decimal digits

   (a) end with an even digit?

   (b) do not contain the same digit twice?

(c) have exactly 3 digits that are 9s?

10. How many strings of 8 letters are there

    (a) if letters can be repeated?

    (b) if letters cannot be repeated?

    (c) that start with $x$, if letters can be repeated?

    (d) that start with $x$, if letters cannot be repeated?

11. How many functions are there from set $\{1, 2, \ldots, n\}$ to $\{0, 1\}$?

12. On each of the 22 work days of a particular month, every employee of a company was sent a company communication. If a total of 4642 total company communications were sent, how many employees does the company have, assuming that there were no staffing changes duing that month?

13. In how many ways can a photographer at a wedding arrange six people in a row from a group of ten people, where the bride and groom are among those ten people, if

    (a) the bride must be in the picture?

(b) both the bride and the groom must be in the picture?

(c) exactly one of the bride and the groom must be in the picture?

14. Suppose that $p$ and $q$ are two different prime numbers and that $n = pq$. Use the inclusion-exclusion principle to find the number of positive integers not exceeding $n$ that are relatively prime to $n$.

15. Determine the number of matches played in a single-elimination tournament with $n$ players, where for each game between two players, the winner goes on, but the loser is eliminated.

16. A bowl has 10 red balls and 10 blue balls. A woman selects balls at random without looking at them.

    (a) How many balls must she select to be sure of having at least 3 blue balls?

    (b) How many balls must she select to be sure of having at least 3 balls of the same color?

17. Let $n$ be a positive integer. Show that among any group of $n+1$ (not necessarily consecutive) integers thare are two with exactly the same remainder when they are divided by $n$.

18. A particular row of Pascal's triangle contains the following numbers.

1 10 45 120 210 252 210 120 45 10 1

What is the next row of Pascal's triangle?

19. What is the coefficient of $x^7$ in the expansion of $(1 + x)^{11}$?

20. How many ordered pairs of integers $(a,\ b)$ do you need to select to guarantee that there are two selected ordered pairs $(a_1,\ b_1)$ and $(a_2,$

$b_2$) such that $a_1 \equiv a_2 \pmod 5$ and $b_1 \equiv b_2 \pmod 5$.

21. Show that if there are 101 people of different heights standing in a line, it is possible to find 11 people in the order they are standing in the line with heights that are either increasing or decreasing.

22. (a) Show that if seven integers are selected from the first ten positive integers, there must be at least be at least two pairs of those

integers that sum to 11.

(b) Is the conclusion of part (a) true if six integers are selected rather than seven?

23. Assuming that nobody has more than 1,000,000 hairs on his or her head and the population of New York City was 8,537,673 in 2016, show that there had to be at least nine people in New York City in 2016 with the same number of hairs on their heads.

24. A particular computer network consists of six computers. Each computer is directly connected to at least one of the other computers.

Show that there are at least two computers in the network that are directly connected to the same number of other computers.

25. What are the values of each of the following?

    (a) $\binom{5}{1} =$

    (b) $\binom{5}{3} =$

    (c) $\binom{8}{4} =$

26. How many bit strings of length 12 contain

    (a) exactly three 1s?

    (b) at most four 1s?

(c) at least four 1s?

(d) an equal number of 0s and 1s?

27. A coin is flipped eight times where each flip comes up either heads or tails. How many possible outcomes

   (a) are there in total?

   (b) contain exactly three heads?

   (c) contain at least three heads?

   (d) contain the same number of heads and tails?

28. How many bit strings of length ten have

   (a) exactly three 0s?

   (b) more 0's than ones?

   (c) at least seven 1s?

   (d) at least three 1s?

29. How many ways are there for ten women and six men to stand in a

line so that no two men stand next to each other?

30. How many ways are there for three penguins and six puffins to stand in a line so that

    (a) all puffins stand together?

    (b) all penguins stand together?

31. Thirteen people on a softball team show up for a game.

    (a) How many ways are there to choose ten players to take the field?

(b) How many ways are there to assign the ten positions by selecting players from the 13 people who show up?

(c) Of the 13 people who show up, three are women. How many ways are there to choose ten players to take the field if at least one of those players must be a woman?

32. Show that if $p$ is a prime number and $k$ is an integer such that $1 \leq$

; $k \leq p$ then $p$ divides $(\binom{p}{k})$.

33. How many different bit strings can be formed using six 1s and eight 0s?

34. How many ways are there to choose a dozen donuts from the 21 varieties at a donut shop?

35. How many different combinations of pennies, nickels, dimes and quar-

ters can a piggy bank contain if it has 20 coins in it?

36. How many ways are there to distribute 12 indistinguishable balls into six distinguishable bins?

37. How many different strings can be made from the letters in MISSISSIPPI using all of the letters?