11. a0 = 3

a1 = 2

a2 = 6

a3 = 8

a­n = 5an-2 - 4an-4 for n > 3

The characteristic equation is r4 – 5r2 + 4 = 0.

The solutions are r = 1, -1, 2, -2.

The general solution is an = c + d(-1)n + e2n + f(-2)n

We need to find c, d, e and f.

a2 = 6 = c + d + 4e + 4f

a0 = 3 = c + d + e + f

Subtract:

3 = 3e + 3f

1 = e + f

a3 = 8 = c – d + 8e – 8f

a1 = 2 = c – d + 2e – 2f

Subtract

6 = 6e – 6f

1 = e – f

So e = 1, f = 0.

a0 = 3 = c + d + 1

a1 = 2 = c - d + 2

Subtract

1 = 2d – 1

d = 1

c = 1

Solution to recurrence:

a­­n = 1 + (-1)n + 2n

12. an = 2an-1 + 2n

a) Show that an = n2n is a solution to this recurrence. We need to show that

n2n = 2((n-1)2n-1) + 2n.

But

2((n-1)2n-1) + 2n = 2n2n-1 – 2(2n-1) + 2n

= n2n – 2n + 2n

= n2n

b) To get all solutions, add a general solution to the associated homogeneous recurrence

an = 2an-1

The characteristic equation is r – 2 = 0.

The general solution is c2n.

So the general form of a solution to recurrence

an = 2an-1 + 2n

is

a­n = n2n + c2n.

c) Suppose a0 = 2.

a0 = 2 = (0)20 + c20

= c

So the solution is a+n = n2n + 2n+1.