11. a0 = 3

 a1 = 2

 a2 = 6

 a3 = 8

 a­n = 5an-2 - 4an-4 for n > 3

 The characteristic equation is r4 – 5r2 + 4 = 0.

 The solutions are r = 1, -1, 2, -2.

 The general solution is an = c + d(-1)n + e2n + f(-2)n

 We need to find c, d, e and f.

 a2 = 6 = c + d + 4e + 4f

 a0 = 3 = c + d + e + f

 Subtract:

 3 = 3e + 3f

 1 = e + f

 a3 = 8 = c – d + 8e – 8f

 a1 = 2 = c – d + 2e – 2f

 Subtract

 6 = 6e – 6f

 1 = e – f

 So e = 1, f = 0.

 a0 = 3 = c + d + 1

 a1 = 2 = c - d + 2

 Subtract

 1 = 2d – 1

 d = 1

 c = 1

Solution to recurrence:

 a­­n = 1 + (-1)n + 2n

12. an = 2an-1 + 2n

a) Show that an = n2n is a solution to this recurrence. We need to show that

 n2n = 2((n-1)2n-1) + 2n.

But

 2((n-1)2n-1) + 2n = 2n2n-1 – 2(2n-1) + 2n

 = n2n – 2n + 2n

 = n2n

b) To get all solutions, add a general solution to the associated homogeneous recurrence

 an = 2an-1

 The characteristic equation is r – 2 = 0.

 The general solution is c2n.

 So the general form of a solution to recurrence

 an = 2an-1 + 2n

 is

 a­n = n2n + c2n.

c) Suppose a0 = 2.

 a0 = 2 = (0)20 + c20

 = c

So the solution is a+n = n2n + 2n+1.