CSCI 3650 Solutions

January 22 Exercises

3. Note, two copies of 41 (41' and 41'')

Initially

41' | 31 26 59 58 41

Insert 31

31 41' | 26 59 58 41'' (1 Shift)

Insert 26

26 31 41' | 59 58 41'' (2 Shifts)

Insert 59

26 31 41' 59 | 58 41'' (No Shifts)

Insert 58

26 31 41' 58 59 | 41'' (1 Shift)

Insert 41''

26 31 41' 41'' 58 59 | (2 Shifts)

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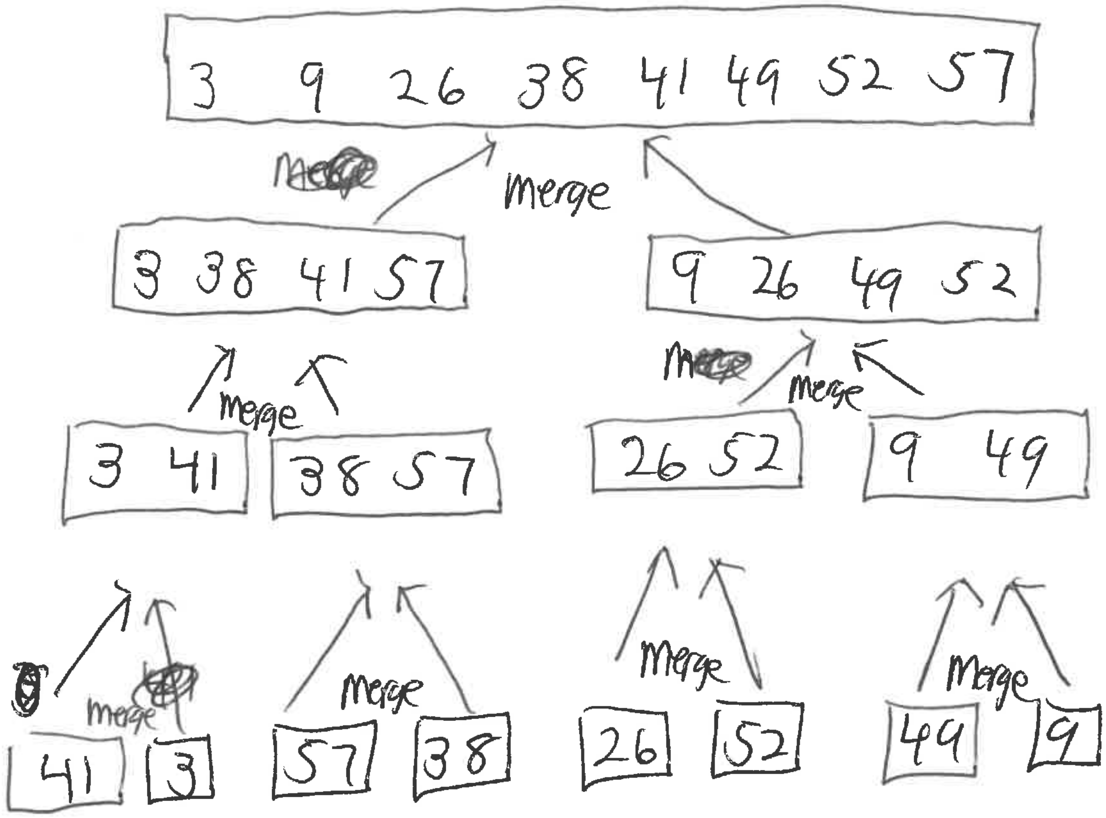
5.



7. Show $\sum_{j=2}^{n}(j-1) = \frac{n(n-1)}{2}$

$$\sum_{j=2}^{n}(j-1) = \sum_{j=2}^{n} j - \sum_{j=2}^{n} 1 = \frac{n(n+1)}{2} - 1 - (n-1)$$

$$= \frac{n(n+1)}{2} - 1 - n + 1 = \frac{n(n+1)}{2} - n \quad \text{(1's cancel)}$$

$$= \frac{n(n+1) - 2n}{2} = \frac{n^2+n-2n}{2} = \frac{n^2-n}{2} = \frac{n(n-1)}{2}$$

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8. $f(n) = \Omega(g(n))$ means

(1) $f(n) \geq c_1 g(n) \geq 0$, $\forall n \geq n_1$ ← constant

$g(n) = \Omega(h(n))$ means

(2) $g(n) \geq c_2 h(n) \geq 0$ $\forall n \geq n_2$ ← constant

Let $n_0 = \max(n_1, n_2)$

then

$f(n) \geq c_1 (\underbrace{c_2 h(n)}_{\text{from (2)}}) \geq 0$ $\forall n \geq n_0$

$f(n) \geq \underbrace{c_1 c_2}_{\text{constant}} h(n) \geq 0$ $\forall n \geq n_0$

$\therefore$ $f(n) = \Omega(h(n))$

10. a) $0 \leq f(n) \leq c\, g(n)$ $\forall n \geq n_0$ $\Rightarrow$ $g(n) \geq \frac{1}{c} f(n) \geq 0$ $\forall n \geq n_0$

b) and c) can both be disproven by same counter example

Let $f(n) = n$, Let $g(n) = n^2$